

Comment on “Growth Inside a Corner: The Limiting Interface Shape”

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In a recent letter, Olejarz et. al.[1] conjecture the asymptotic shape of a crystal, grown by depositing cubes inside a three dimensional corner, by generalizing the known two dimensional results consistent with the symmetries of the problem. The conjecture differs from numerical simulations by 0.9%, but the discrepancy is ascribed to a slow approach to the asymptotic answer. We do Monte Carlo simulations which avoid these transients and conclude that the conjecture is inconsistent with our numerical results.

The growth model in Ref. [1] is a solid on solid (SOS) model where the integer height $z(x, y)$ at a lattice point (x, y) is constrained by $z(x, y) \leq \min[z(x-1, y), z(x, y-1)]$, with the boundary conditions $z(x, y) = \infty$ if $x < 0$ or $y < 0$. At time $t = 0$, $z(x, y) = 0$ for $x, y \geq 0$. With rate 1, $z(x, y)$ increases by 1 provided the new configuration is a valid one. The conjecture for the asymptotic shape, when projected along the $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ direction, reduces to $x = y = z = wt$, where $w = 1/8$.

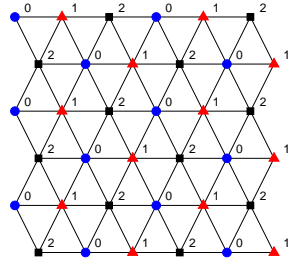


FIG. 1: On projecting the surface of the crystal onto the $(1, 1, 1)$ plane, a triangular lattice is obtained. If h_i is the height at a site on sublattice i , then $h_i \bmod 3 = i$. In addition, $|h_i - h_j| < 3$ for all nearest neighbor pairs $\langle ij \rangle$.

The growth velocity in any direction is a function of the local slope $\partial z/\partial x$ and $\partial z/\partial y$, and is best determined using a different set of boundary conditions when these slopes are uniform everywhere [2]. Thus, for measuring w , we project the growing crystal onto the $(1, 1, 1)$ plane to obtain a restricted solid on solid (RSOS) model on a triangular lattice, to which we apply periodic boundary conditions. The heights on sublattice i of the triangular lattice (see Fig. 1) is i modulo 3. In addition, there is a constraint $|h_m - h_n| < 3$ for nearest neighbor sites m, n . With rate 1, the height at a site increases by 3 provided the new configuration satisfies the RSOS condition. The periodic boundary conditions restores translational in-

variance, and we measure in the steady state the fraction of sites that can increase in height. It is straightforward to see that this fraction is identical to w .

We consider systems of size $L \times L$ where $L = 30 \times 2^n$, with $n = 0, \dots, 5$. The steady state averaged data for w is shown in Fig. 2, where the error in each data point is estimated by doing many independent runs. We estimate $w = 0.12606(2)$, different from the conjectured value $w = 1/8 = 0.125$. While our estimate for w is consistent with the simulation results in Ref. [1], the discrepancy from $1/8$ can no longer be ascribed to transients. We also observe that w reaches its asymptotic value as $L^{-\theta}$ with $\theta \approx 1.25$ (see inset of Fig. 2).

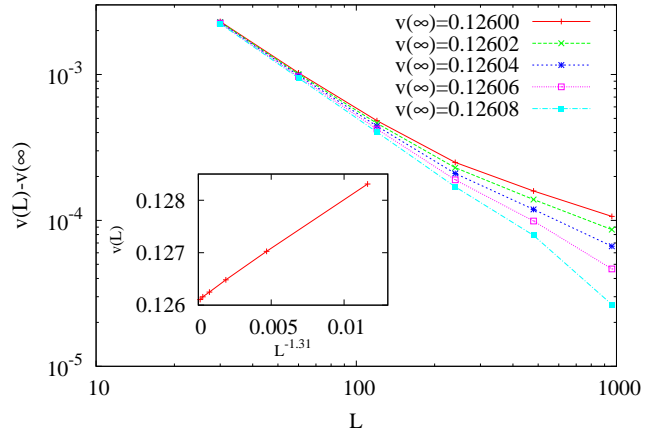


FIG. 2: The difference between the velocity $w(L)$ and the asymptotic result $w(\infty)$ as a function of system size L for different $w(\infty)$. The curve is roughly a straight line for $w(\infty) = 0.126065$. For smaller (larger) $w(\infty)$, the data curve upwards (downwards). Inset: $w(L)$ against $L^{-1.25}$ is a straight line with intercept larger than 0.126.

It may be that the asymptotic shape is a linear combination of the two solutions outlined in Ref. [1]. If that is the case, the relative weights have to be shown to be independent of $\partial z/\partial x$ and $\partial z/\partial y$. This is again best demonstrated on a triangular lattice with appropriate boundary conditions.

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